Rochester Institute of Technology
Manufacturing & Mechanical Engineering Technology

DEVELOPMENT OF AN EMPIRICAL THEORY

Determining Damping Coefficient with respect to Mass and Inertia

MCET – 400 – 01
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1.0 EXECUTIVE SUMMARY

Recently, Zephyr Power Systems Inc. (ZPS Inc.) has expanded their product line into consumer wind turbines. These smaller turbines are meant to be robust and be resilient against mechanical ware and part failure. To avoid this, ZPS Inc. wants to remove the mechanical brake and implement a damping vane that acts as a wind brake. Unfortunately, they did not have a means of determining the size of their damping vane. This led them to contact GD & T – REX, and request an analysis on the relationship between moment of inertia (MOI), surface area of damper (A_S), and the resulting damping coefficient, 'α.'

GD & T – REX was not able to find a mathematical solution to this situation, and resorted to empirical testing to find the relationship. Through testing a pendulum, GD & T – REX was able to simulate the damping of motion of a rotating object. Through the damping of this harmonic oscillation, and the variation of this experiment's independent variables, damping coefficients could be found. Five different moments of inertia were tested against 11 different surface areas of damping vanes. Collecting data produced by the rotational motion of the pendulum allows for an exponential decay curve to be fit to the peaks of the rotation (when the instantaneous rotational velocity of the pendulum is zero.) The exponential decay curve that is fit to the curve has a constant of damping. This damping constant was extracted and saved into the table below.

Surface Area [m^2]	Rotational Moment of Inertia [kg*m^2]				
	0.00299	0.00841	0.01507	0.02440	0.04444
0	0.033	0.014	0.007	0.005	0.003
0.003226	0.119	0.092	0.054	0.034	0.019
0.006452	0.227	0.164	0.097	0.065	0.035
0.009677	0.359	0.221	0.130	0.090	0.052
0.01290	0.476	0.283	0.153	0.114	0.063
0.01613	0.634	0.345	0.204	0.138	0.078
0.01935	0.738	0.404	0.226	0.160	0.090
0.02258	0.885	0.512	0.287	0.197	0.101
0.02581	1.003	0.611	0.341	0.202	0.115
0.02903	1.134	0.721	0.389	0.238	0.131
0.03226	1.349	0.803	0.418	0.245	0.141

Table 0: Array of damping coefficients.

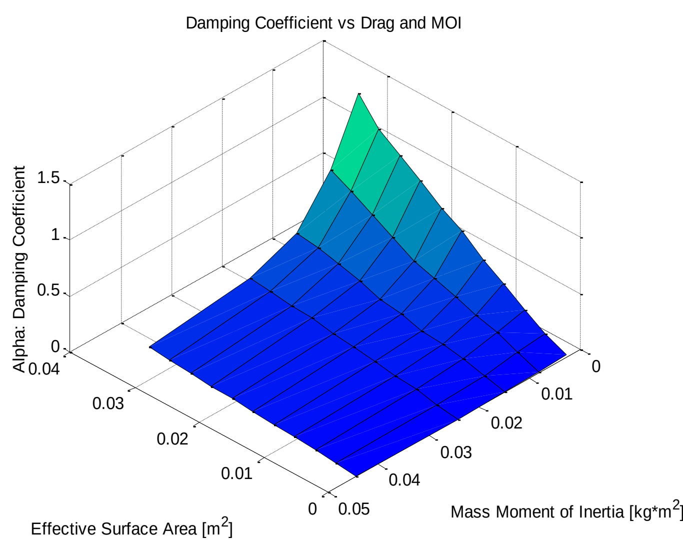


Figure 0: Surface plot of damping coefficients.

The table (above left) is an empirically derived chart that can be used by ZPS Inc. to approximate the correct damper size when the moment of inertia and necessary damping coefficient are known. This chart does allow for linear interpolation between numbers, and the surface plot (above right) shows the mathematical relationship between the moment of inertia, effective surface area, and resulting damping coefficient.

While interpolation is a decent method of approximation between the collected points in table 0, extrapolation may not be. This experiment had a secondary goal of covering the broadest range of MOIs and A_Ss as possible. While there may be a trend inside the range of collected data, there may not be the same trend outside of the collected data. GD & T - REX does not recommend extrapolation of this data.

2.0 INTRODUCTION

2.1 PROJECT DEFINITION AND PLAN

Zephyr Power Systems Inc. (ZPS Inc.) is developing a line of small wind turbines for off-grid power generation. For safety reasons, the speed of the rotation must be controlled. The lack of control would lead to rotational over-speed, and subsequently catastrophic turbine blade failure. The present over-speed prevention mechanism is a mechanical brake that engages when turbine speeds approach dangerous levels. Unfortunately, this brake is mechanically complex and wears out quickly in windy locations, posing an ongoing maintenance issue for customers. ZPS Inc. is exploring the possibility of an air resistance damper or vane, which is likely to wear out much slower than a mechanical brake, if at all.

Since ZPS Inc. markets several sizes of wind turbine, the damping vane will need to vary in size depending on the speed and size of each particular wind turbine. Because there is no *simple* physics-based method of predicting damping coefficient as a function of size and moment of inertia, these values will need to be empirically derived. GD & T-REX has been hired to perform measurements of damping effectiveness and create a model allowing ZPS Inc. to pick the necessary damper vane size based of factors such as desired damping coefficient α, mass of turbine blade, and moment of inertia of the blade.

During the preparation of this experiment, there were multiple factors to keep in mind. Initially, there was a daunting task of analyzing all of the data across many variables. This also posed the problem of running out of time due to the high and possibly unnecessary quantity of tests. Because of this, two variables were isolated to test, rather than three or more. Moment of inertia is compared against effective surface area of the damping vane and then analyzed to produce a damping coefficient for each combination of the two independent variables. These variables were chosen due to two simple facts. First, the entire purpose is to create an air resistance based damper. Air resistance is, for calculation purposes, related to the largest cross sectional area of a body perpendicular to its motion through the air. Thus, changing the surface area of this experiment was a necessity. Second, there is a property of a rotating object that associates the mass of the object with the required torque to change the motion of the object. This property is known as the rotational moment of inertia. This characteristic is important because ZPS Inc. wants to apply this force to slow the motion of the object. The moment of inertia of the propeller would be an initial condition chosen by the company, and, through the company's calculations, they would also choose a necessary damping coefficient. From these two choices, ZPS Inc. is then able to find the surface area of the corresponding damping vane needed to preserve the mechanical integrity of their windmill products.

To test these variables, a pendulum with attachable masses (to change rotational moment of inertia) and swappable damping cards (to change air resistance) is used. The rotational positions can be recorded as stated later in section 4.1. The method of analysis for determining the damping coefficient is listed in section 4.2.3.

3.0 Description of Test

3.1 Apparatus Sketch

Three main constructions were used in this experiment. First was the mechanism provided to each team. The mechanism consisted of three parts. First, the rotational recorder and bare pendulum were provided. Second, the pendulum arm was attached to the rotational recorder and was rigidly attached via screw to the rotational portion of the rotational recorder. Third, the pendulum arm ended in an attachment with the ability to hold a set of damping vanes. This setup is attached rigidly to the test frame, as denoted by the datum shown on Figure 1 below. The damping vane had the ability to be attached (as shown by dotted lines) in a friction fit within the plastic adapter. Lastly, the break shown on the pendulum arm denotes a 90-degree rotation of the arm along its axis so that the visual can include both the rotational axis as well as the effective surface area of the damping card.

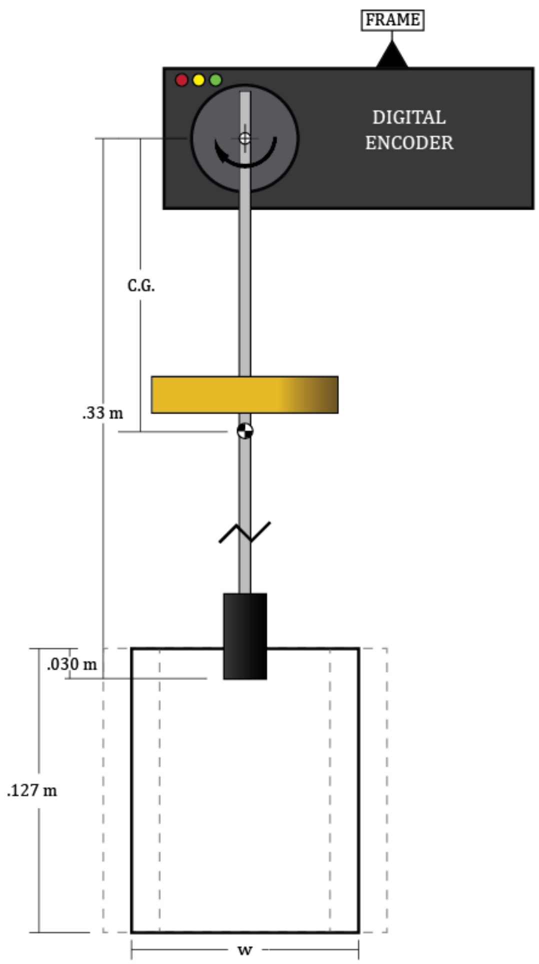


Figure 1 - Apparatus sketch of pendulum setup

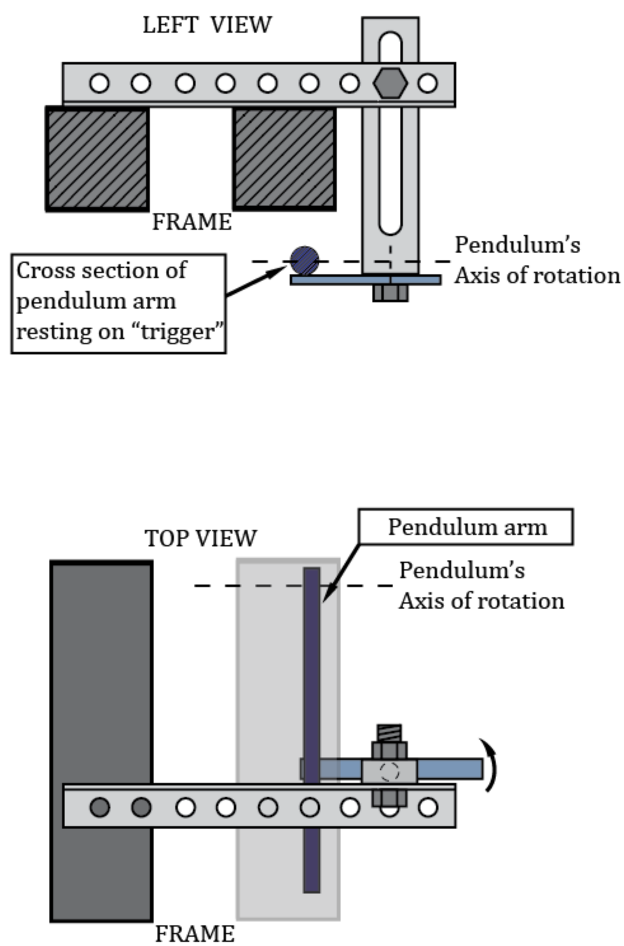


Figure 2 - Custom pendulum securing device

A second mechanical device was custom made by a team member of GD & T-REX. Shown above in Figure 2, there is a trigger type mechanism (light blue) that locks the pendulum (dark blue) to a perfectly horizontal position until released. This allows for a consistent start from rest for every repetition of the experiment.

Lastly, a custom set of damping vanes was built for this experiment. They will be referred to as damping cards from this point forward. The first damping card is a rectangle with dimensions 12.7 cm by 2.54 cm in height and width respectively. Each of the following 9 cards are an additional 2.54 cm in width to the previous rectangle. *(Shown below: Figure 3)* They are made in such a way that they all have the same mass, and same horizontal center of gravity. Having a consistent mass and consistent center of gravity allows for the approximation of constant moment of inertia as the experimental damping card changes in size. Because of this, more accurate computations can be completed in section 4.2. A further in-detail description of the purpose of these custom damping cards is listed in the commentary of section 4.2.3 as well.

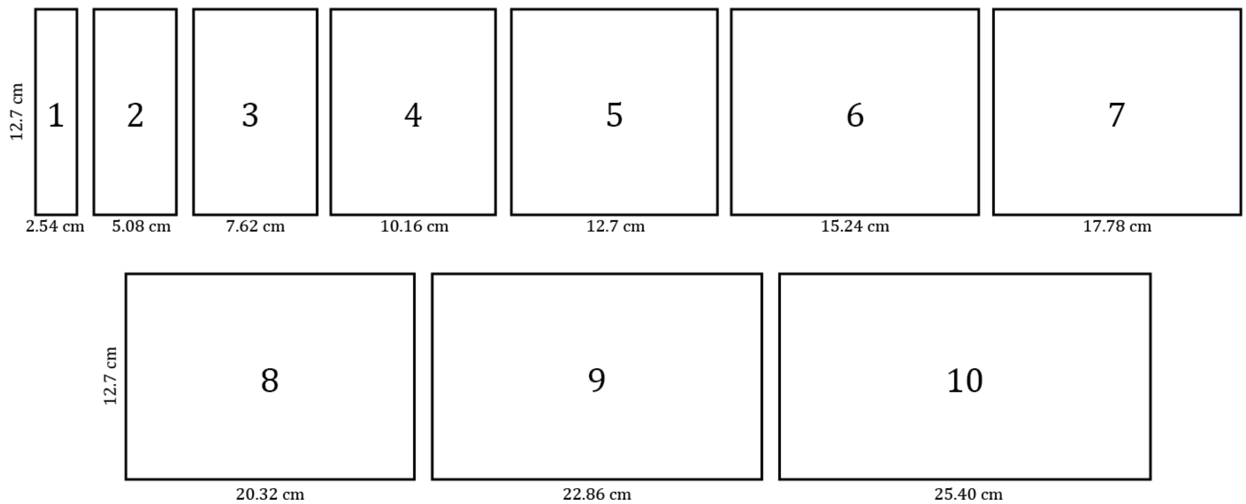


Figure 3 - Damping Cards - Height is a constant 12.7 cm and width is the number on the cards multiplied by 2.54 cm

Each of these apparatuses were each utilized multiple times during the experiment and maintained consistency throughout all of the tests. Solid models of any of these devices are obtainable per request.

3.2 RELATED THEORY

The data had a certain damping coefficient that dictated how quickly the set of data traveled to zero. The goal of finding this number is to relate the effective surface area to how quickly the angular velocity of the pendulum changes. There is a process to derive the differential equation that provides this solution. Below is a discussion on the derivation of the equation used to fit the curves in this experiment.

The equation is presented in differential calculus. Covered briefly in the Mechanical Dynamics class, the equation for damped motion comes from Newton's second law. The sum of the forces acting on this pendulum are modeled to a certain extent by the same equation. The original differential equation takes into account the force due to damping, in this case, the force due to air resistance . Solving this differential equation, which can be found online [Gilchrist,1], provides the general case for underdamped motion. This general case will be used to approximate the motion in this situation. Shown below is the general form, along with a visual representation *(Figure 4)* of undamped motion.

$$\theta(t) = \theta_o e^{-\alpha t}[\cos(\omega t - \beta)] \qquad (1)$$

General form for underdamped rotational motion.

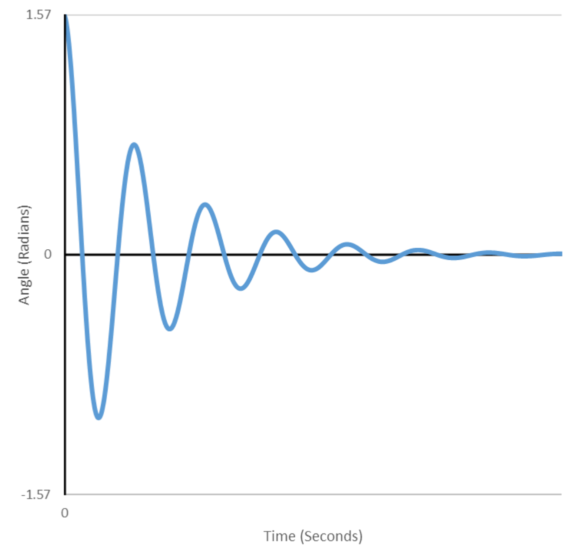


Figure 4 – Sample Plot of Underdamped Oscillation

The above picture illustrates the theoretical motion of the pendulum, where the y-axis represents the angle, and the x-axis represents time. This motion has a certain decay rate shown by the change in amplitude of each oscillation as a function of time. This decay rate is known as the envelope curve, and is actually a component of the above equation (1). Shown below is the general form for equation (2); the envelope curve, as well as the visual representation of how the envelope curve interacts with the distinct motion curve.

$$\theta(t) = \pm\theta_o e^{-\alpha t} \qquad (2)$$

General form for envelope curve.

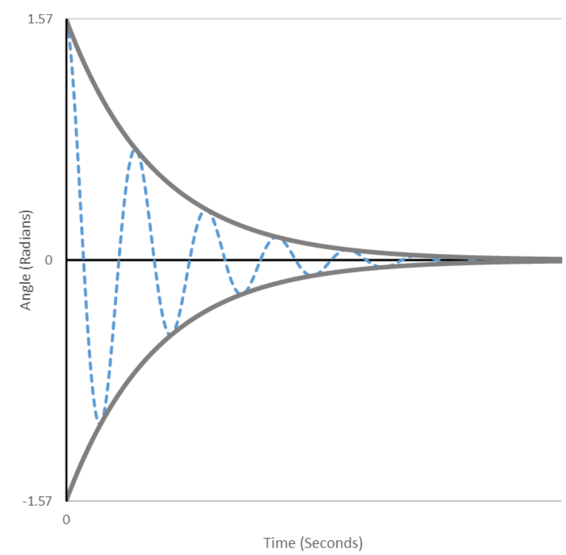


Figure 5 - Envelope Curve for Underdamped Oscillation

With the knowledge of this envelope curve, which is only an exponential curve, the data can be fit to this function. Fitting the data to the function using MATLAB will provide an accurate estimate of the damping coefficient α as long as the R^2-value is reasonably high. For this experiment, the R^2-value must be above .9 as decided by the team. This seemed like a good cutoff number because of the expectation that the model explains at least 90% of all variability in the collected points around the fit curve.

4.0 DATA ACQUISITION AND ANALYSIS

4.1 DATA ACQUISITION; METHODS USED

Conducting this experiment was a predetermined process. The week before, the team created an instructional packet outlining the process used to test and collect data. The instruction sheet, attached in Appendix A is abbreviated and discussed in this section.

Initially, GD & T-REX started by measuring all of the mechanical equipment for mass and size. Using a pair of dial calipers, the pendulum arm was measured for length and diameter. The masses were then recorded in an excel spreadsheet and referenced for calculations in section 4.2. Next, the true size and mass measurements of the custom made damping cards were completed. Measurements and masses were recorded for the purpose of calculating moment of inertia and effective surface area for air resistance.

The experiment has a secondary goal of capturing the widest range of data possible. To achieve this, eleven different iterations of different surface areas and five iterations of different moment of inertias were used. Starting, the empty pendulum with no added masses or damping cards was attached to the digital encoder. The purpose of the digital encoder was to record the angles of the pendulum 50 times a second during each test. From this point the computer was connected to the digital encoder and LabVIEW was used to read and record the data. A file provided by RIT's MMET Department named "Pendulum2.vi" was utilized to record the data in specific. This program allowed for a zero point to be set. The digital encoder was rigidly attached to the pendulum, and the pendulum was allowed to hang freely so that gravity was the influencing force to keep the arm vertical. When the arm had come to rest the program was set to zero.

The arm was then rotated clockwise to a perfectly horizontal position (90°) and was locked into the apparatus described in section 3.1. The LabVIEW file was then run, and the pendulum was dropped within one or two seconds of the program starting. The LabVIEW program collected the data which was then exported to a Microsoft Excel document.

Next, the smallest size damping card was attached to the pendulum arm. Keeping all other factors the same, the step in the paragraph above was repeated. This repetition occurred a total of 11 times. Once for no additional damping card, and then with damping cards 1 to 10. Each of the cards had an increase of 32.26 cm^2 (5 in^2) more than the card before it.

After testing once with no damping card, and one with every other card, the pendulum was carefully removed and a brass mass was added to the pendulum. This is the first increase in moment of inertia of this body. The brass weight is positioned 12 cm from the axis of rotation, and then tested through the steps outlined in the above two paragraphs. After these tests have been completed and recorded, the weight is moved to 18 cm, then 24 cm and tested in the same way. Lastly, a second brass weight is added to the 23* cm position. This increases the moment of inertia to the maximum that can be achieved in this testing environment. The tests are run as stated before, and the data recorded and exported to excel.

**Second brass weight was stacked on the first one at the 24 cm position, and the brass weight is 1cm thick.*

In total, there were 11 changes in surface area, and 5 changes in moment of inertia. Each combination of surface area and moment of inertia produced a damping coefficient from the set of pendulum oscillations collected. The process used to convert this oscillation to a single damping coefficient is outlined in the following section.

4.2 DATA ANALYSIS; SAMPLE CALCULATIONS

4.2.1 FBD's and Sketches

The accompanied sketch below is a graphic visual to demonstrate the calculation of the moment of inertia. The calculation, which references these figures, can be seen in Part 1 of section 4.2.3. The break shown on the pendulum arm in subfigure Ⓣ is a 90-degree rotation along the shaft. Like before, this is done to show the face of the damping card as well as the rotational axis.

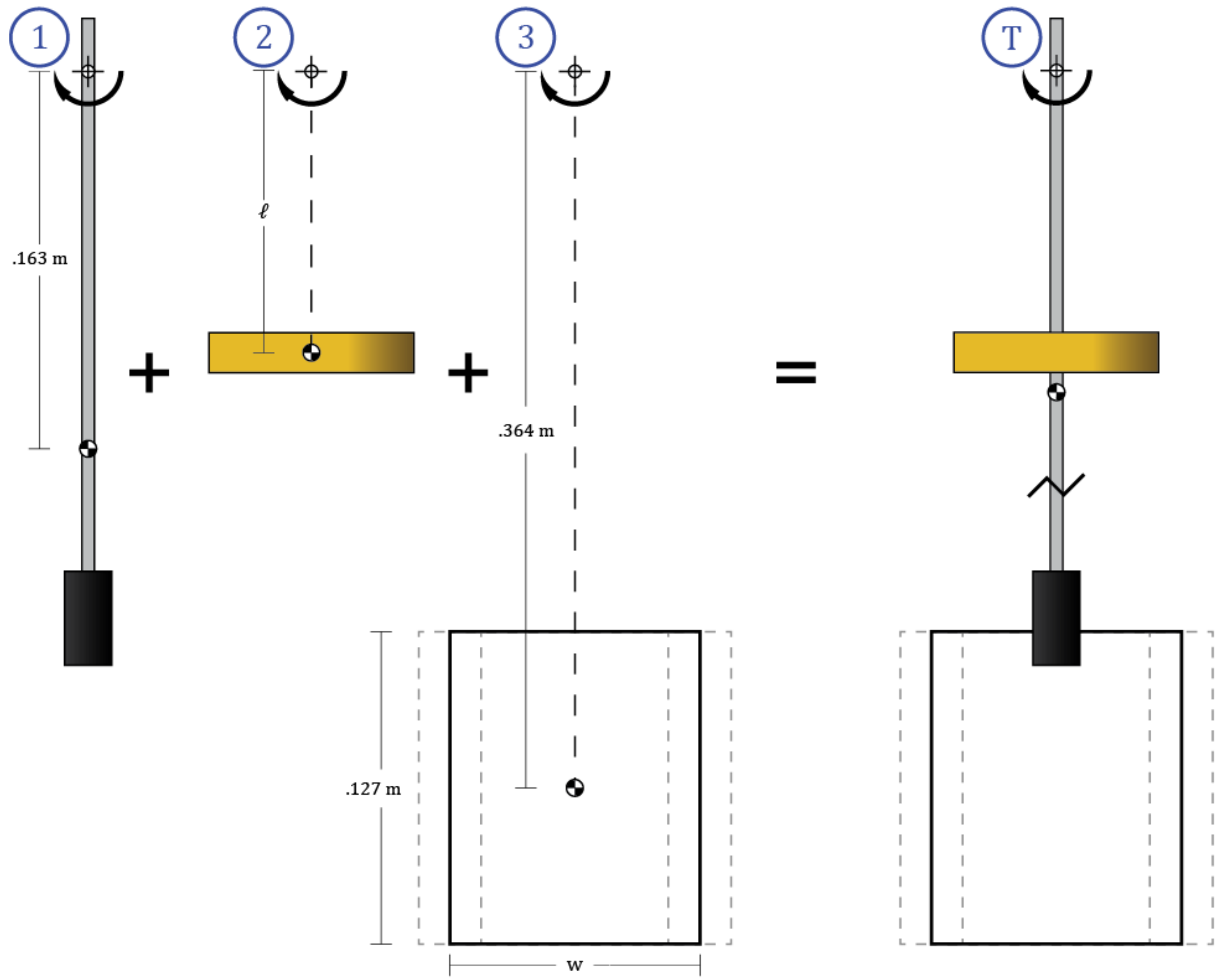


Figure 6 - Visual representation of Parallel Axis Theorem for sample moment of inertia calculation

This experiment does not require any further free body diagrams. This solution has already been provided for the differential equation that results from analyzing these forces (Section 3.2).

4.2.2 Variable Identification

Two main types of variables were used in this experiment. The first were measured values conducted with either dial calipers or a triple beam balance. The second were independent variables which were purposely changed to get a reaction from the dependent variable.

Measured Variables:

- Brass Weight 1:
  - Diameter: 7.00 cm
  - Thickness: 1.00 cm
  - Mass: 307.0 g
- Brass Weight 2:
  - Diameter: 7.00 cm
  - Thickness: 1.00 cm
  - Mass: 307.7 g
- Pendulum Arm:
  - Diameter: .63 cm
  - Length: 33 cm
  - Mass: 37.5 g
  - Balance: 18.8 cm from top end
- Damping Card "n" where n is 1 to 10:
  - Height: 12.70 cm
  - Width: n*2.54 cm
  - Mass: 9.5 g

Hypothesized Variables:

- Independent:
  - A_S – Normal-to-motion surface area of the damping card
  - I_{nYY} – Mass moment of inertia of the pendulum
- Dependent
  - α – Damping coefficient due to wind resistance and angular velocity

These variables cover most of what needs to be tested. Programmatic variables are made for the purpose of conversions and comparisons in Part 2 of section 4.2.3, but those variables do not belong in this section because of their transient nature.

4.2.3 Calculations

Two stages of calculations were conducted for this experiment. The first stage was the preliminary calculations, mainly used to identify the values used as categorical variables for the second stage of the calculations. The second stage is the actual analysis of the damping due to air resistance.

PART 1:

The categorical variables can be defined as the independent variables of this test. In theory, both of these independent variables are continuous but, for testing purposes, are made into categorical style predictors with values. The first category is that of the damping cards. The 10 cards had the goal of capturing the smallest to largest range possible. This range was chosen to fit the available cardstock sheets of one of the team members. There were 11 even increments that started at “no card attached” to “nearly full sheet of paper.” The sheets were reinforced with lightweight pieces of wood in an evenly distributed fashion. This allowed the area to increase with negligible change to the vertical center of gravity and thus the moment of inertia of the pendulum. Reinforcing the paper also provided for easier calculations. Without the added rigidity, the paper would bend as the force of air resistance increased. As the paper bent, the effective surface area would change, and this step would be a differential equation rather than a constant scalar area. The calculation of the area of the damping cards is shown below. Recall that the effective surface area (A_S) is the area normal to the motion of the object. In this case, it would just be the area of the rectangle of the card ‘n.’

$$A_S = h * w$$

$$A_S = 12.70[cm] * 2.54n[cm]$$

$$A_S = .1270[m] * .0254n[cm]$$

For damping card n = 1

$$A_S = .1270[m] * .0254(1)[cm]$$

$$A_S = .003226\ [m^2]$$

This calculation is repeated for n = 1:10. The results are shown in the second column of table 4.

The second of these categories is moment of inertia (MOI) of the rotating body. Five different increments between the smallest and largest possible values were calculated. The first increment did not include a brass weight. To show the full sample calculation, the second increment is used. Figure X is indicative of the following calculation, and effectively shows a visual representation of the parallel axis theorem. A sample calculation is shown below:

> First, it is necessary to find the rotational mass moment of inertia about the object’s center of mass. The pendulum arm was a composite shape that could not be done with a simple equation. To avoid error, the part was measured with dial calipers and created in SolidWorks. When finished, the center of gravity and mass were almost no different from the measured values. This can be seen in Appendix B. SolidWorks reported the mass moment of inertia for the pendulum arm to be:
>
> $$I_{①yy} = 3.897 * 10^{-4}[kg \cdot m^2]$$
>
> For the brass disc, this axis would be the one that would project out of the page in Figure 6. This inertial property is given by the formula below.
>
> $$I_{②yy} = \frac{m}{12}(3r^2 + h^2)$$
>
> Plugging in values specific to this problem yields:

$$I_{②yy} = \frac{307.0[g]}{12}(3(3.5[cm])^2 + (1[cm])^2)$$

Converting to standard base unit values:

$$I_{②yy} = \frac{.3070[kg]}{12}(3(.0350[m])^2 + (.0100[m])^2)$$

$$I_{②yy} = 9.658 * 10^{-5}[kg \cdot m^2]$$

Next, the mass moment of inertia was calculated for the damping card. This axis of rotation would run horizontal across the page (through the center of gravity) in figure 6. The formula below provides the means to find the mass moment of inertia of this shape. Notice it does not include the width as denoted in figure 6. This is by design, as the cards can now change width without influencing the moment of inertia. It is understood that all the cards have the same thickness (depth, 'd') of .35 cm and the same mass of 9.5 g. The cards were balanced as best as possible, but we still need to make the assumption that the cards have a constant density and a center of mass located directly in the middle.

$$I_{③yy} = \frac{m}{12}(d^2 + h^2)$$

Plugging in values specific to this problem yields:

$$I_{③yy} = \frac{9.5[g]}{12}((.3[cm])^2 + (12.7[cm])^2)$$

Converting to standard base unit values:

$$I_{③yy} = \frac{.0095[kg]}{12}((.0030[m])^2 + (.1270[m])^2)$$

$$I_{③yy} = 1.278 * 10^{-5}[kg \cdot m^2]$$

Lastly, the parallel axis theorem states that these three inertias are rotating about a single, parallel axis. The equation that results from this theorem is below. The ℓ in the following equations refer to the difference in length of the parallel axis to the center of mass of each object.

$$I_{Ⓣyy} = \sum(I_{ⓝyy} + m_n {\ell_n}^2)$$

Expanding this summation for ①, ②, and ③, provides:

$$I_{Ⓣyy} = \left(I_{①yy} + m_1 {\ell_1}^2\right) + \left(I_{②yy} + m_2 {\ell_2}^2\right) + (I_{③yy} + m_3 {\ell_3}^2)$$

Plugging in known values: ℓ_2 is 12 cm for this case.

$$I_{Ⓣyy} = (3.897 * 10^{-4}[kg \cdot m^2] + (.0375[kg])(.188[m])^2) +$$

$$(9.658 * 10^{-5}[kg \cdot m^2] + (.3700[kg])(.120[m])^2) +$$

$$(1.278 * 10^{-5}[kg \cdot m^2] + (.0095[kg])(.364[m])^2) =$$

$$I_{Ⓣyy} = .008411[kg \cdot m^2]$$

This process is repeated for each change in moment of inertia. The other values are present in the first numerical row of table 4.

The categorical calculations are now completed and there is a precise coordinate at which a damping coefficient can be assigned. Think of this as the base of a 3D plot. The floor (x,y-plane) would be defined as the moment of inertia along the x axis (on which there are 5 instances) and the effective surface area of the damping cards along the y-axis (on which there are 11 instances). This creates an array of 55 points; at each point a z-value, or damping coefficient, is recorded. This gives depth to the x,y-plane and creates a 3D plot of the data. Part 2 covers the calculation of the damping coefficient.

PART 2:
After each test, the data was temporarily stored in the LabVIEW file. This data was then exported into excel in an organized manner. Every change in moment of inertia warranted a new excel document; every change of damping card warranted a new sheet within each excel document. On each sheet there were two columns: one for time, and one for recorded angular position. GD & T-REX determined the analysis of this data by hand was not an effective use of time, and thus resorted to a program called MATLAB for data analysis. MATLAB provides the ability for software to complete the same task repeatedly as fast as the computer allows. After writing the script discussed below, all of the data was analyzed within two minutes. The script, attached in Appendix B, is outlined below using pseudo-code and a logical progression of mathematics.

The first task was to be able to successfully find and read the data from excel. First, all five excel documents were placed into the Documents/MATLAB folder so that it may be found when called for. The command "`xlsfinfo()`" was used to obtain information about the excel file, such as the number of sheets. The command "`xlsread()`" was used to read the numbers within each sheet and save it as an array with two columns of numbers. This initialized the set of data that was to be processed to find the damping coefficient.

From here, the mathematical operations began. First, the first column of X data was converted to time, and stored as variable 't' as shown in table 1 below. This was done by dividing the time by 50, as there were 50 samples per second for all the tests. This conversion was applied to all the X data but only ten samples are shown.

X Data	Conversion	t (seconds)
0	÷50	0
1	÷50	0.02
2	÷50	0.04
3	÷50	0.06
4	÷50	0.08
5	÷50	0.1
6	÷50	0.12
7	÷50	0.14
8	÷50	0.16
9	÷50	0.18
10	÷50	0.2

Table 1 – X Data to time conversion.

Next, the Y data was observed. The digital encoder records the angular position in 360 increments, corresponding with the degrees in a circle. Unfortunately, degrees are not actually numbers, and thus we convert to radians to form useable data. This conversion from degrees to radians is simple: multiply by π and divide by 180. Table 2 below demonstrates this conversion for ten samples.

Y Data	Conversion	θ (radians)
90	*(π/180)	1.5708
90	*(π/180)	1.5708
90	*(π/180)	1.5708
90	*(π/180)	1.5708
89	*(π/180)	1.5533
82	*(π/180)	1.4312
74	*(π/180)	1.2915
62	*(π/180)	1.0821
48	*(π/180)	0.8378
32	*(π/180)	0.5585
15	*(π/180)	0.2618

Table 2 – Y Data to radians conversion.

The next step was to determine the time of the drop. The LabVIEW program was started one or two seconds before the pendulum arm was dropped. This resulted in a string of identical data in the Y data while the X data was changing. The goal of this next operation is to identify the point at which the pendulum is dropped and set that point as the starting time. To do this, a portion of the MATLAB script tested if a Y value was less than a previous one. If the Y value was less than the previous one, it would mean the pendulum was starting to fall. If not, the script would test the next value. When it identified this value, it would save it as "t_start" and break the loop. This portion of the script is shown below.

```
for j = 1:length(RawData(:,2)         %start counting through sheet to find
                                      initial drop location.
    if theta(j) < 90                  %if the starting value is less than 90,
       t_start(i) = j-1;              %record the point into a value t_start for
                                      that sheet.
         break                        %After that, break the loop to stop
                                      recording start values.
    end                               %only the time value of the first point is
                                      saved.
end
    time = (1/samples persecond)*(t - t start(i))    %offset time array
```

Figure 7 - MATLAB script excerpt for t_start identification

From there, the variable created in table 1 needs to be modified to accompany the new start time. Removing the initial offset removes error introduced when the curve fitting function tries to fit a function without an offset to the present function with an offset. Shown below is a table outlining the adjustments made to the data above.

i	t (seconds	θ (radians)	t_start	time	θ (radians)
1	0	1.5708	FALSE	-0.06	1.5708
2	0.02	1.5708	FALSE	-0.04	1.5708
3	0.04	1.5708	FALSE	-0.02	1.5708
4	0.06	1.5708	0.06	0	1.5708
5	0.08	1.5533	-	0.02	1.5533
6	0.1	1.4312	-	0.04	1.4312
7	0.12	1.2915	-	0.06	1.2915
8	0.14	1.0821	-	0.08	1.0821
9	0.16	0.8378	-	0.1	0.8378
10	0.18	0.5585	-	0.12	0.5585
11	0.2	0.2618	-	0.14	0.2618

Table 3 – Y Data to radians conversion.

Demonstrated above, as the iteration 'i' progresses, the script tests the θ values for change. When it detects a change, it records the 't' and creates a new variable 'time' to act as the new string of data. After t_start is used to remove the time offset 't' had, the new data is saved into an array with columns "time" and "θ;" the last two columns are almost ready to be analyzed for peaks.

One more simple operation was applied to the θ values before the finding the peaks. The data from the lowest moment of inertia condition did not have many peaks, and thus a wide confidence interval was present. In order to obtain more maximums, the absolute value of the θ data was taken. Visually, the original graph shown in figure 8 is converted to figure 9. This conversion allows for more points to be collected as peaks, which narrows the confidence interval of α. The absolute value operation conducted on θ is repeated for all future θ values as well. In MATLAB the command for this is "`abs(theta)`." This method does introduce a small amount of error (discussed in section 5.0), but this error is resolved by the smaller confidence intervals around α from MATLAB.

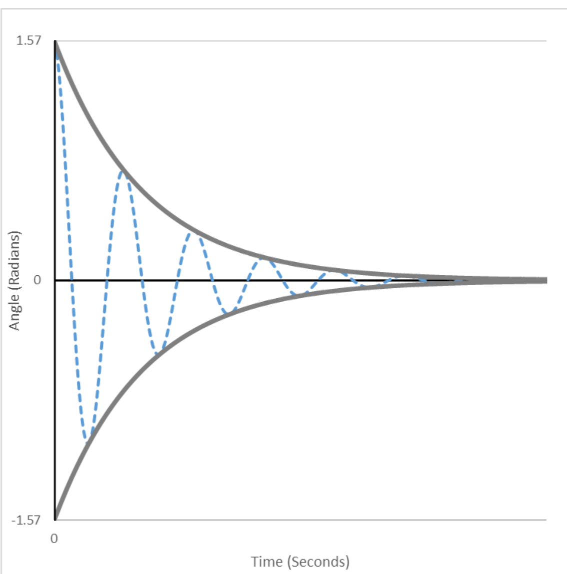


Figure 8 - Plot of oscillation with envelope curve

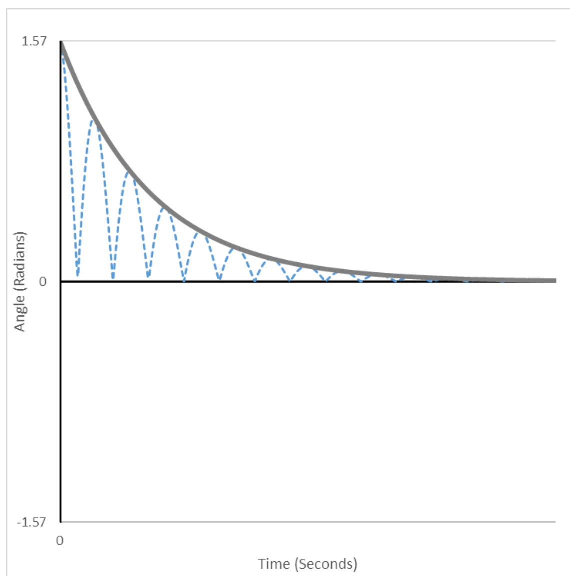


Figure 9 - Plot of absolute value of oscillation with envelope curve

Analyzing the data for the peaks was as simple as a modified logic test to the one listed in figure 7. The logic starts by knowing the θ value at the start time. The MATLAB script tests the value of the θ value with the surrounding θ values. If θ at a certain point is a number 'a,' then the script tests the previous value and the value after. If 'a' is greater than both, it is recorded as a maximum. If there is a plateau in the data and the values on either side of 'a' are equal to 'a' the script reaches out two values to either side of 'a.' This may continue for as many points are included in the plateau. If the plateau occurs at a maximum, the first point is included in the array of maximums. This method does not work for data with collection noise, as it identifies relative maximums and would identify the small maximums within the collection noise. Luckily, the data acquisition method listed in section 4.1 was a very stable, repeatable process. This prevented additional data smoothing that may have changed the values of the data to an unacceptable range. The maximums of the data in the array "`[time, θ]`" were run through this process. The maximum θ values along with the time at which they happen are recorded into a new array, "`[a, b]`" where 'a' are the peaks and 'b' are the corresponding times. The function that performs these tasks is known as "`findpeaks(theta)`" where the script written by GD & T-REX calls for this function, rather than copying the function into one script.

MATLAB can then use the Least Squares Regression (LSQR) method to fit the array of peaks and times to an exponential curve. Documentation on the method of an LSQR fit to an exponential curve is found in Weisstein, *(Wolfram)*. The function used to fit the array [a, b] is called by the command "`fit(x,y,exp1)`." The x data requested by this command correlates to the variable named "b," and the y data correlates to the variable named "a." The third field in this command asks for the type of curve this fit attempts. In this case we are fitting a single exponential decay function listed in section 3.2 "*General form for envelope curve.*" The output of the "`fit()`" command are the coefficients "θ" and "α" as well as a 95% confidence interval for α and an R^2 value for curve-fit quality. The output appears as shown below:

```
[f,g] = fit(b,a,'exp1')

f =
             General model Exp1:
             f(x) = c*exp(d*x)
             Coefficients (with 95% confidence bounds):
                    c =       1.498      (1.45, 1.547)
                    d =      -0.1143     (-0.1217, -0.1068)
g =
           sse: 0.0287
       rsquare: 0.9841
           dfe: 18
    adjrsquare: 0.9833
          rmse: 0.0399
```

Figure 10 -MATLAB output for fit coefficients and R2 value.

Shown above in figure 10, the key values for the collected data are shown in blue. The first, "d," is the damping coefficient, and the second, "rsquare" is the coefficient of determination. In section 3.2 it is discussed that, for this experiment, an R^2 value over .900 makes for a valid fit. For this set of data, the curve is a good fit, and the α value (shown above as 'c') can be regarded as accurate. Figure 11 shows the MATLAB plot of this data. The fit for one set of data (one moment of inertia and one card) is shown as a red line, where the θ points are shown in blue.

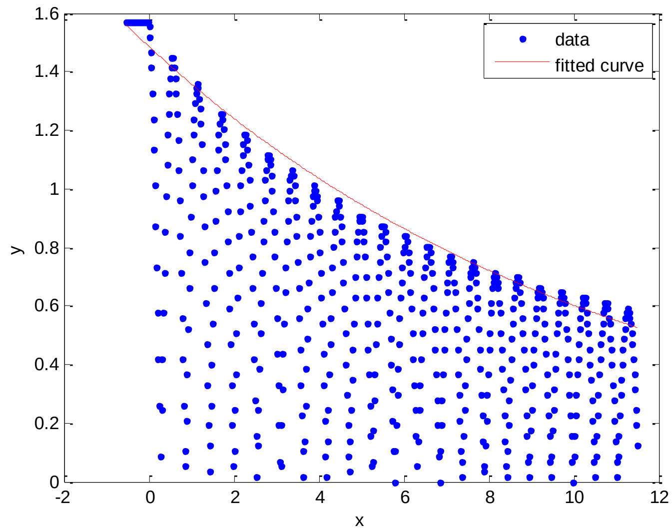


Figure 11 - MATLAB plot of θ (y) vs time (x)

This process was repeated for all 55 tests. Again, the full MATLAB code is included in Appendix B, as well as an array of plots with fit lines similar to the one above in Appendix C.

4.3 Data Products

4.3.1 Graphs

Choosing the graphs can be a bit misleading. The two plots below point out a reason why there might be some misinterpretation on a mathematical basis based on the plot a user may initially view.

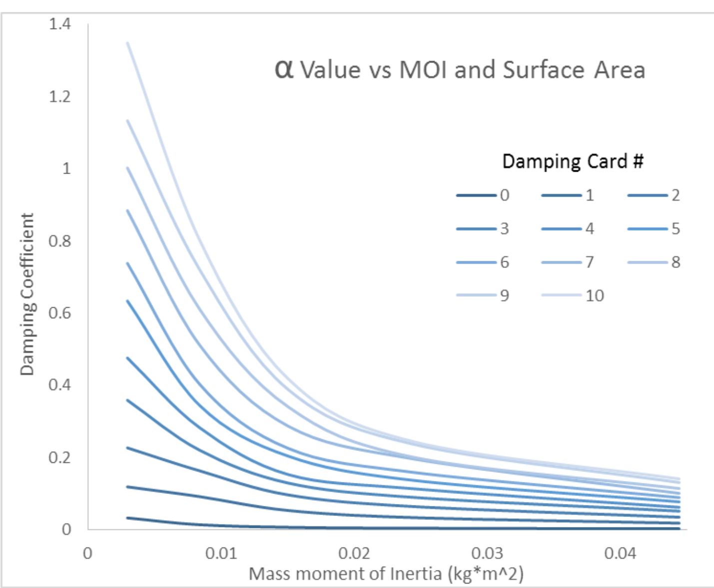


Figure 12 - Contour Curves of Surface Area

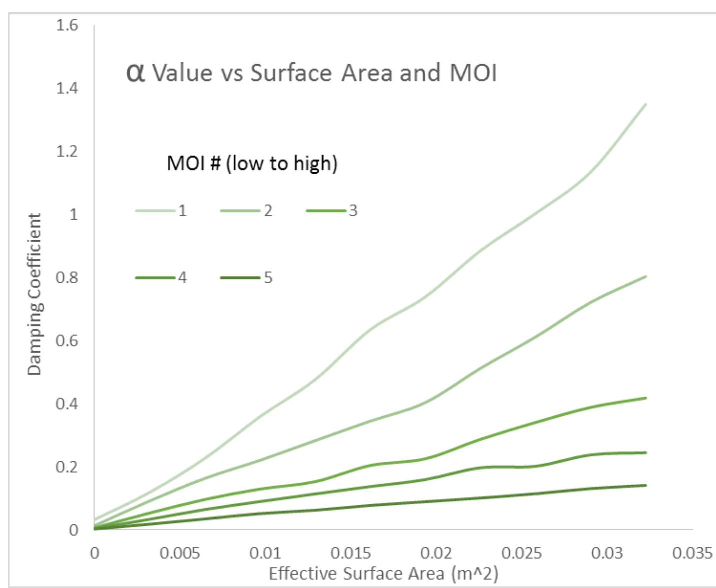


Figure 13 - Contour curves of Moment of Inertia

Initially, a reader may be drawn towards Figure 12 because of the smooth curves, but this set of contour curves make it very difficult to list on the plot the surface area. Because of the dense population of these curves, there is very little room to include the surface areas at either end of these curves. This plot would have to be accompanied by a sort of table that includes the surface area represented by each line on this plot. This may cause a reader to be drawn towards the second plot, figure 13. There are only five separate lines and each of them have plenty of room to list a true numerical value for moment of inertia. Unfortunately, linear interpolation would not be an accurate method of finding middle values. Looking at figure 12, it is blatantly obvious that the differences between the lines is non-linear. Due to this, figure 13 would be misleading to include in documentation because it leads the reader to believe that there is no issue with linear interpolation between each of the recorded moment of inertia lines. To avoid both of these issues, a 3D surface plot can be used to provide numerical scales for both the variables without sacrificing the readability of the damping coefficient values.

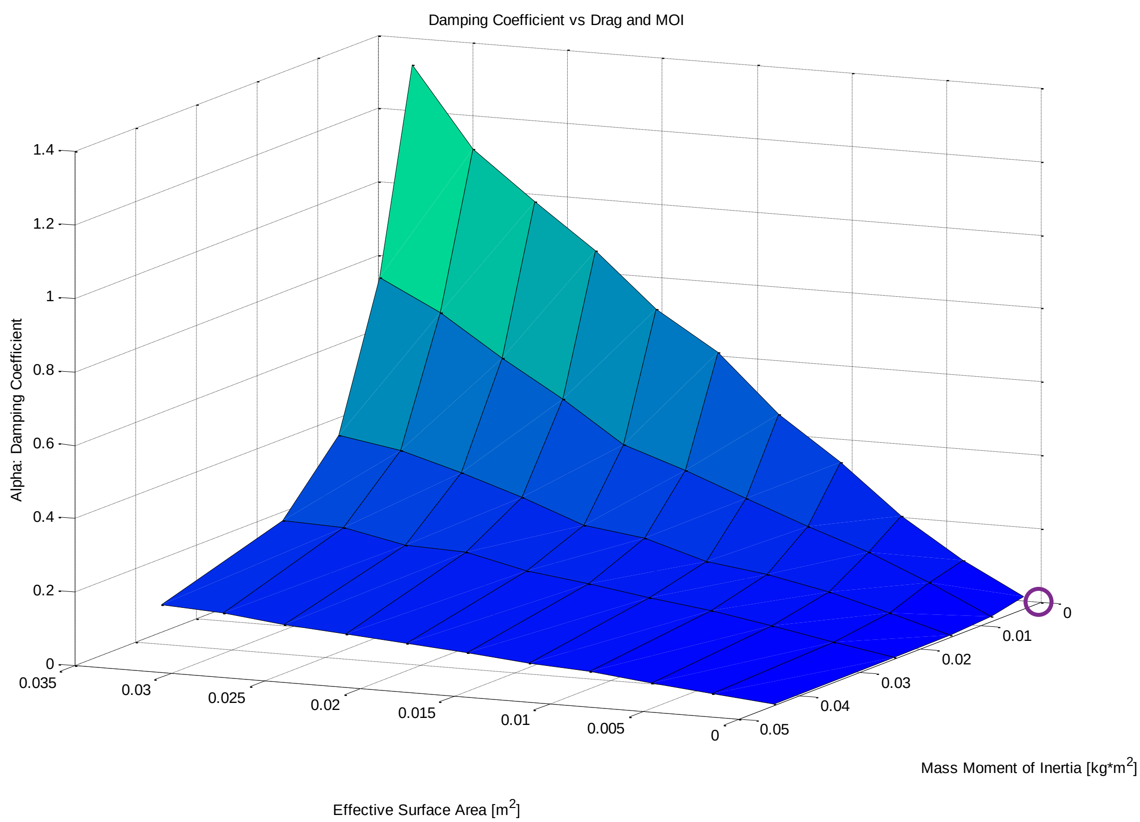


Figure 14 - 3D Surface plot of damping coefficient as a function of Surface area and moment of inertia

Figure 14 is oriented in a fashion to make it easier to perceive the curvature of this surface. It shows the curvature present along the moment of inertia – damping plane, and the linearity present along the surface area – damping plane. The true origin of the plot shown is located at the purple circle in the bottom right corner. This plot has the ability to be interpolated anywhere along its surface and provides for an easier understanding for those with spatial reasoning.

4.3.2 Tables

The table used to produce the 3D plot is labelled below as "table 4" and holds the 55 damping constants obtained at each intersection of the 11 damping card conditions and the 5 moment of inertia conditions.

Inertial Condition		MOI 1 [kg*m^2]	MOI 2 [kg*m^2]	MOI 3 [kg*m^2]	MOI 4 [kg*m^2]	MOI 5 [kg*m^2]
Card #	A_s [m^2]	0.00299	0.00841	0.01507	0.02440	0.04444
0	0	0.033	0.014	0.007	0.005	0.003
1	0.003226	0.119	0.092	0.054	0.034	0.019
2	0.006452	0.227	0.164	0.097	0.065	0.035
3	0.009677	0.359	0.221	0.130	0.090	0.052
4	0.01290	0.476	0.283	0.153	0.114	0.063
5	0.01613	0.634	0.345	0.204	0.138	0.078
6	0.01935	0.738	0.404	0.226	0.160	0.090
7	0.02258	0.885	0.512	0.287	0.197	0.101
8	0.02581	1.003	0.611	0.341	0.202	0.115
9	0.02903	1.134	0.721	0.389	0.238	0.131
10	0.03226	1.349	0.803	0.418	0.245	0.141

Table 4 – List of Surface areas vs MOI's and corresponding α value.

This table is also another useful tool that can be used by engineers. The uses and operations of this table are further discussed in Section 6 of this report.

Another table that is important to the findings is the coefficient of determination table. This table includes a graphic of the fit and an R^2 value at every location there is an α (white cell). This table is included in Appendix C attached at the end of this document. The R^2 values are all above .9, meaning the least squares regression line (exponential fit model) explains more than 90% of the variation in the data for all of the 55 cases. The significance of this is also discussed in section 6.

The other relevant tables that show progression of data analysis are included in section

5.0 Measurement Error Table and Analysis

Measurement Tool	Accuracy
Dial Calipers:	0.00002 m
Metric Ruler:	0.001 m
Triple Beam Balance:	0.1 g
Digital Encoder:	1°

Table 5 – Measurement tools and associated gage error.

The most critical set of measurements conducted were those included in the moment of inertia calculations. This was because the standard equations for inertia usually have terms that are squared ore sometimes cubed. This means any error included in the original measurement goes from being insignificant to rather large. A similar idea is shown in Appendix B in “Calculation for MOI 2” and “Calculation for MOI 3.” The calculated moment of inertia 3 is 179% greater than the calculated moment of inertia 2. Initially this seems too drastic to be true, but upon inspection, the only term to change between these two calculations is the ℓ_2 value. This value increases from .12 m to .18 m between the two calculations. The six one-hundredths change in the distance nearly doubled the calculated MOI. This difference shows that the distance from rotational center is arguably the most critical measurement and any error present in this measurement is now squared, significantly increasing the data to noise ratio.

There was some error mathematically introduced into the analysis: taking the absolute value of the curve flipped the series of data that originally only had ±1° of possible error due to the ability of the digital encoder. Now, by choosing to take the absolute value of this set of data, the possibility of error doubles to ±2°. This was later accounted for by the confidence/tolerance interval settings in MATLAB. More information on this can be obtained by request. The slight introduction of error was expected, and helped with many of the highly-damped scenarios, but it always seems contradictory to make the conscious decision to introduce error.

With that being said, many precautions were taken to avoid other types of error. The first of these precautions is listed in section 3.1. The locking mechanism that released the pendulum from the same height removed the possibility of the pendulum starting at different angles. The reinforced damping cards also prevented the paper from bending and kept the effective surface area the same throughout the pendulum’s whole swing. Doing this kept the independent variable of surface area steady through the entire test, rather than having it change slightly as a function of radial velocity. Also, a pair of metric calipers was used to take measurements where possible, rather than converting to inches. Lastly, the damping cards had negligible mass by comparison to some of the larger structures on the rotating body of the pendulum. Despite this, it was taken into account and was found to have a significant impact on the moment of inertia of the body. Each of these precautions were beneficial to keeping a tight precision as well as high accuracy in this experiment.

6.0 Discussion of Results and Conclusion

This experiment was well prepared for and encountered almost no roadblocks along the way. The day of the test provided fully functioning equipment with no technical errors. All mechanical attachments were secure, and the frame’s wheels were locked so no counter oscillations contributed to the damping constant.

Later, while analyzing the data, there seemed to be a slight issue that was not expected in the collection of the data. Although the instruction sheet in Appendix A listed a specific time that the LabVIEW file was supposed to record for, the team realized this was not always enough time to capture the decay, or was too much time to provide an accurate curve. There seems to be a “sweet spot” for the amount of time the program records. If the time is stopped too early, there is not enough data to pull peaks from, and the data tends to fall in a linear pattern, which does not yield a ‘α’ value. If the collection happens for too long, the pendulum falls from the transient state where it decreases exponentially to the steady state where it oscillates and the damping effect is negligible. The steady state provides a lot of data outside of the range where its oscillations decay exponentially. This changes the fit coefficient. Luckily, none of the collected data reached too far into either of these cases, but if this experiment would be repeated, it would be wise to set the recording interval to record until the pendulum has decayed a certain amount rather than for a set time. Due to this being a purely empirical test, there is little theory to compare to the obtained data. Any theoretical relations are covered in section 3.2.

The main table of results, a modified version of table 4, is provided as the most condensed set of results from this experiment. This table, table 6, can be used much like fluid sciences uses empirically derived values for their science. These results will be transferred to Zephyr Power Systems Inc. as reference material as they produce their new line of off-grid power generation turbines. If Zephyr Power Systems Inc. knows the rotational inertia of their product and the necessary amount of damping to maintain the integrity of their product, this table can identify a reasonable value for damping vane size. This will save money in the long run on repairs, maintenance, and replacements of their products. The table below can be used to linearly interpolate values as well, as long as the value th in question is reasonably close to one of the adjacent values.

Surface Area [m^2]	Rotational Moment of Inertia [kg*m^2]				
	0.00299	0.00841	0.01507	0.02440	0.04444
0	0.033	0.014	0.007	0.005	0.003
0.003226	0.119	0.092	0.054	0.034	0.019
0.006452	0.227	0.164	0.097	0.065	0.035
0.009677	0.359	0.221	0.130	0.090	0.052
0.01290	0.476	0.283	0.153	0.114	0.063
0.01613	0.634	0.345	0.204	0.138	0.078
0.01935	0.738	0.404	0.226	0.160	0.090
0.02258	0.885	0.512	0.287	0.197	0.101
0.02581	1.003	0.611	0.341	0.202	0.115
0.02903	1.134	0.721	0.389	0.238	0.131
0.03226	1.349	0.803	0.418	0.245	0.141

Table 6 – condensed results for damping coefficient in tabular form.

It was mentioned that there may be the ability to extrapolate from this data as well, but this is not necessarily a safe idea. The experiment was designed to obtain the widest possible range of moments of inertia as well as the widest possible change in surface area without overdamping the system. Predicting data outside of this testing region is almost asking for something to go wrong. There may be evidence that there is a trend inside the tested region but this does not mean the trend continues outside of the specified region. *This is the disclaimer that extrapolation may not be a good idea*. Despite this, interpolation is a completely acceptable method of *approximation* and can be used within this table or accompanied surface plot.

7.0 APPENDICES

REFERENCES

Gilchrist,1, Alexei. "Damped Harmonic Oscillator." *Damped Harmonic Oscillator*. N.p., 2014. Web. 22 Mar. 2016. <http://www.entropy.energy/scholar/node/damped-harmonic-oscillator>.

Gilchrist,2, Alexei. "Driven and Damped Oscillator." *Driven and Damped Oscillator*. N.p., 2014. Web. 25 Mar. 2016. <http://www.entropy.energy/scholar/node/driven-damped-oscillator>.

Weisstein, Eric W. "Least Squares Fitting--Exponential." -- *from Wolfram MathWorld*. Wolfram Alpha, n.d. Web. 14 Mar. 2016.<http://mathworld.wolfram.com/LeastSquaresFittingExponential.html>.

APPENDIX A

GD & T-REX Instruction Packet:

PROBLEM STATEMENT:

Zephyr Power Systems Inc. (ZPS Inc.) is developing a line of small wind turbines for off-grid power generation. For safety reasons, the speed of the rotation must be controlled. Lack of control would lead to rotational over speed, and subsequently catastrophic turbine blade failure. The present over speed prevention mechanism is a mechanical brake that engages when turbine speeds approach dangerous levels. Unfortunately, this brake is mechanically complex and wears out quickly in windy locations, posing an ongoing maintenance issue for customers. ZPS Inc. is exploring the possibility of an air resistance damper or vane, which is likely to wear out much slower than a mechanical brake, if at all.

Since ZPS Inc. markets several sizes of wind turbine, the damping vane will need to vary in size depending on the speed and size of each particular wind turbine. Because there is no simple physics-based method of predicting damping coefficient as a function of size and moment of inertia, these values will need to be empirically derived. GD & T-REX has been hired to perform measurements of damping effectiveness and create a model allowing ZPS Inc. to pick the necessary damper vane size based on factors such as desired damping coefficient α, mass of turbine blade, moment of inertia of the blade, and other such factors.

EQUIPMENT:

- Digital Encoder
- Brass weights
- Triple beam balance
- Meter stick
- Dial calipers
- Cardstock cards ranging from 5in^2 to 50in^2 (32.26cm^2 to 322.58cm^2)
  Note: Each card has a width of 5in (12.7cm)
- Custom release Mechanism
- Computer with LabVIEW software and Excel software

PROCEDURE: (USE METRIC – CGS - UNITS THROUGHOUT)

Part 1 – Setup / Calibration of Equipment

1. Gather all equipment and properly zero the dial calipers and the triple beam balance.
2. Measure and record the mass of the pendulum and the brass weights using the triple beam balance.

3. Measure the diameter and the thickness of the brass weights using the dial calipers.
4. Locate the center of gravity of the pendulum using the knife edge tool.
5. Measure the distance from the center of gravity of the pendulum to the center of the top bolt hole on the pendulum arm using the calipers.
   Note: The top bolt hole will be the pivot point of the pendulum.
6. Attach the pendulum to the digital encoder wheel using the top bolt hole on the pendulum arm.
7. Make sure that the curved portion of the pendulum arm rests in the grooves on the digital encoder wheel when attaching the pendulum to the wheel.
8. Position the release mechanism so that the pendulum will be rotated 90-degrees from its resting position while it is being held by the release mechanism.
9. Open the LabVIEW file named **Pendulum2**.
10. Using the dropdown menus, set fields in the top left column so that the match the text directly to the right of them.
11. Set the “points/sec” field to **50 points/sec**.

Part 2 – Collection of Raw Data

1. With the pendulum arm at rest in the vertical position, start the LabVIEW program by clicking the “start” arrow in the top left corner of the LabVIEW window.
2. Once the program has started, raise the pendulum arm until it is held by the release mechanism.
3. Release the pendulum and let it swing for 10 seconds.
4. After 10 seconds, click the stop button in LabVIEW, right click on the graph generated by LabVIEW, and export the data to Excel.
5. Right click on the graph again and select “clear chart” to clear the data.
6. Analyze the data and if there appears to be any large gaps/jumps between the points, increase the points/sec to 100 points/sec.
7. Test the pendulum with no cards or weights attached to create a baseline data set.
8. Attach the 50in^2 (322.58cm^2) card to the polymer piece at the end of the pendulum arm and run the test by following steps 1-5.
9. Replace the 50in^2 (322.58cm^2) card with the next smallest card and run the test.
10. Continue this process until all the cards have been tested.
11. Add 1 brass weight with the outer ring to the pendulum arm, 12.5cm from the center of the top bolt hole on the pendulum arm.
12. Run the test on the weighted pendulum with no cards to create a baseline for the weighted pendulum.

13. Repeat steps 8-10 using the weighted pendulum.

14. Move the brass weight added in step 11 so that it is now positioned at the top of the polymer end piece on the pendulum arm.
15. Repeat steps 12 and 13
16. Add another brass weight including the outer ring to the pendulum arm and position it so that it is flush with the first weight.
17. Repeat steps 12 and 13.

REPORT TYPE:

1. Generate an Angle vs Time plot for each testing setup.
2. Calculate all polar moments of inertia for each testing setup (see additional information).
3. Upload Excel data for each testing setup into MATLAB program (see additional information) to calculate damping coefficients.
4. Generate a Damping Coefficient vs Area plot using the polar moment of inertia as the series name, area as the independent variable, and damping coefficient as the dependent variable.

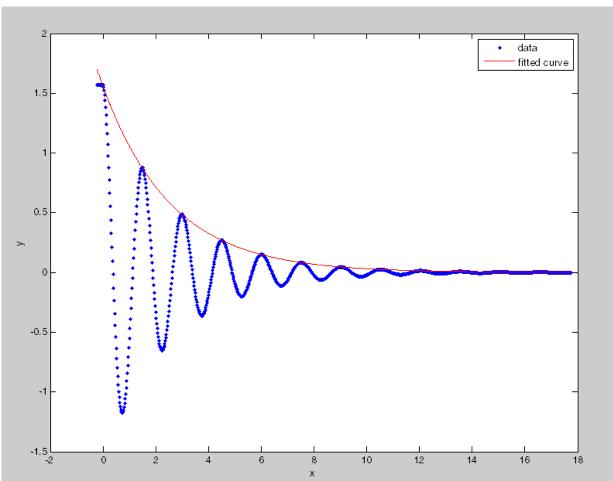
Additional Information

Along with the calculation of damping coefficient, there are a few other calculations that influence the output of this testing method. One of the big influential factors is the moment of inertia of the body as it is swinging. This moment of inertia of the system can be written as a function of the three bodies about the rotational axis; (1) the arm, (2) the brass mass, and (3) the paper damper. This combination would appear as the equation below:

$$I_{Total-A} = \left(I_1 + m_1 {d_1}^2\right) + \left(I_2 + m_2 {d_2}^2\right) + \left(I_3 + m_3 {d_3}^2\right)$$

Originally, the damping panels did not greatly influence the moment of inertia of the system, but the low mass meant low rigidity, and low rigidity meant the surface area of the damping pane would change as it flexed. This would create an issue where surface area is a function of the velocity of the pendulum. Because this would never be the case for a marketed windmill, rigid panes were constructed. They all are specially designed to have equal masses with concentric centers of gravity, and *approximately* equal moments of inertia, so changing the pane does not change the overall MOI.

Lastly, a program has been written to analyze this data, rather than doing repetitive calculations by hand. This program has the ability to identify and isolate the relative maximums, and then fit an exponential decay line to best estimate the enveloping curve of the data. In short, the input would be an excel file of all the exported LabVIEW tests, and **the output would be the damping coefficients**. An example of the fit achieved is shown below:



Field	Value	Min	Max
sse	3.2802e-06	3.2802...	3.2802...
rsquare	1.0000	1.0000	1.0000
dfe	10	10	10
adjrsquare	1.0000	1.0000	1.0000
rmse	5.7273e-04	5.7273...	5.7273...

```
>> alpha

ans =

   -0.3891
```

Fin.

APPENDIX B

Additional Calculations

The calculation for the mass moment of inertia (for the composite pendulum arm *only*) is shown from the SolidWorks assembly file. The "Mass Properties" feature was utilized to compile the mass moment of inertia about its center (Lxx) as well as the mass moment of inertia about the point at which it rotates (Ixx). Because the mass and center of mass align with the measurements taken in section 4.2.2, it is assumed the inertial properties listed below are also correct.

Exported from SolidWorks:

Mass properties of Pendulum Full
Configuration: Default
Coordinate system: Coordinate System1

Mass = 0.03747947 kilograms

Volume = 0.00001897 cubic meters

Surface area = 0.01297130 square meters

Center of mass: (meters)
X = 0.00000000
Y = -0.18712904
Z = -0.00007281

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)
Taken at the center of mass.

Ix = (0.00000000, 0.99999994, -0.00035647)	Px = 0.00000113
Iy = (-0.99999999, 0.00000000, -0.00013302)	Py = 0.00043127
Iz = (-0.00013302, 0.00035647, 0.99999993)	Pz = 0.00043127

Moments of inertia: (kilograms * square meters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00038971	Lxy = 0.00000000	Lxz = 0.00000000
Lyx = 0.00000000	Lyy = 0.00000113	Lyz = -0.00000015
Lzx = 0.00000000	Lzy = -0.00000015	Lzz = 0.00043127

Moments of inertia: (kilograms * square meters)
Taken at the output coordinate system.

Ixx = 0.00171511	Ixy = 0.00000000	Ixz = 0.00000000
Iyx = 0.00000000	Iyy = 0.00000113	Iyz = 0.00000035
Izx = 0.00000000	Izy = 0.00000035	Izz = 0.00171511

Calculation for MOI 1 – No brass disc, includes ① (pendulum arm) and ③ (damping card)

$$I_{Ⓣyy} = \left(I_{①yy} + m_1 \ell_1^{\,2}\right) + \left(I_{③yy} + m_3 \ell_3^{\,2}\right)$$

Plugging in known values: $I_{ⓝyy}$ are calculated in Part 2 of section 4.2.3.

$$I_{Ⓣyy} = (3.897 * 10^{-4}[kg \cdot m^2] + (.0375[kg])(.188[m])^2) +$$

$$(1.278 * 10^{-5}[kg \cdot m^2] + (.0095[kg])(.364[m])^2) =$$

$$I_{Ⓣyy_1} = .002987[kg \cdot m^2]$$

Calculation for MOI 2 – Includes ① (pendulum arm), ② (brass disc), and ③ (damping card)

$$I_{Ⓣyy} = \left(I_{①yy} + m_1\ell_1{}^2\right) + \left(I_{②yy} + m_2\ell_2{}^2\right) + \left(I_{③yy} + m_3\ell_3{}^2\right)$$

Plugging in known values: ℓ_2 is 12 cm for this case.

$$I_{Ⓣyy} = (3.897 * 10^{-4}[kg \cdot m^2] + (.0375[kg])(.188[m])^2) +$$

$$(9.658 * 10^{-5}[kg \cdot m^2] + (.3700[kg])(.120[m])^2) +$$

$$(1.278 * 10^{-5}[kg \cdot m^2] + (.0095[kg])(.364[m])^2) =$$

$$I_{Ⓣyy_2} = .008411[kg \cdot m^2]$$

Calculation for MOI 3 – Includes ① (pendulum arm), ② (brass disc), and ③ (damping card)

$$I_{Ⓣyy} = \left(I_{①yy} + m_1\ell_1{}^2\right) + \left(I_{②yy} + m_2\ell_2{}^2\right) + \left(I_{③yy} + m_3\ell_3{}^2\right)$$

Plugging in known values: ℓ_2 is 18 cm for this case.

$$I_{Ⓣyy} = (3.897 * 10^{-4}[kg \cdot m^2] + (.0375[kg])(.188[m])^2) +$$

$$(9.658 * 10^{-5}[kg \cdot m^2] + (.3700[kg])(.180[m])^2) +$$

$$(1.278 * 10^{-5}[kg \cdot m^2] + (.0095[kg])(.364[m])^2) =$$

$$I_{Ⓣyy_3} = .015071[kg \cdot m^2]$$

Calculation for MOI 4 – Includes ① (pendulum arm), ② (brass disc), and ③ (damping card)

$$I_{Ⓣyy} = \left(I_{①yy} + m_1\ell_1{}^2\right) + \left(I_{②yy} + m_2\ell_2{}^2\right) + \left(I_{③yy} + m_3\ell_3{}^2\right)$$

Plugging in known values: ℓ_2 is 24 cm for this case.

$$I_{Ⓣyy} = (3.897 * 10^{-4}[kg \cdot m^2] + (.0375[kg])(.188[m])^2) +$$

$$(9.658 * 10^{-5}[kg \cdot m^2] + (.3700[kg])(.240[m])^2) +$$

$$(1.278 * 10^{-5}[kg \cdot m^2] + (.0095[kg])(.364[m])^2) =$$

$$I_{Ⓣyy_4} = .024395[kg \cdot m^2]$$

Calculation for MOI 5 – Includes ① (pendulum arm), ② (brass disc), ③ (damping card), and ④ (brass disc 2)

$$I_{Ⓣyy} = \left(I_{①yy} + m_1\ell_1{}^2\right) + \left(I_{②yy} + m_2\ell_2{}^2\right) + \left(I_{③yy} + m_3\ell_3{}^2\right) + \left(I_{④yy} + m_4\ell_4{}^2\right)$$

Plugging in known values: ℓ_2 is 24 cm, and ℓ_4 is 23 cm for this case.

$$I_{Ⓣyy} = (3.897 * 10^{-4}[kg \cdot m^2] + (.0375[kg])(.188[m])^2) +$$

$$(9.658 * 10^{-5}[kg \cdot m^2] + (.3700[kg])(.240[m])^2) +$$

$$(1.278 * 10^{-5}[kg \cdot m^2] + (.0095[kg])(.364[m])^2) +$$

$$(9.680 * 10^{-5}[kg \cdot m^2] + (.3707[kg])(.230[m])^2) =$$

$$I_{Ⓣyy_5} = .044435[kg \cdot m^2]$$

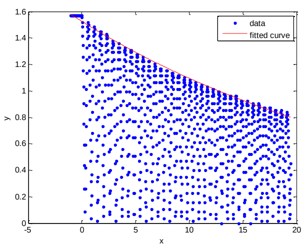
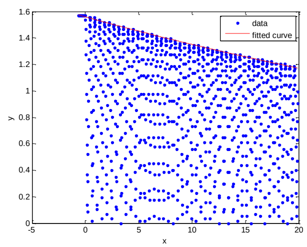
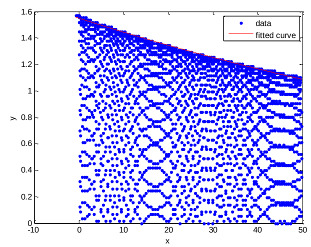
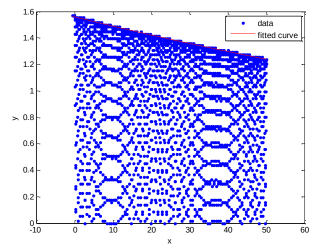
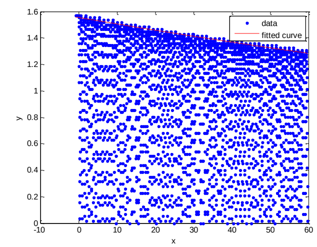
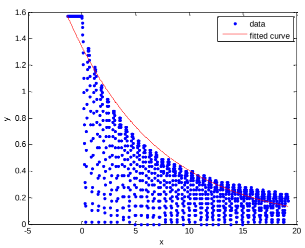
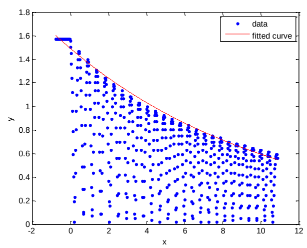
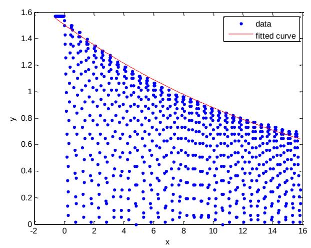
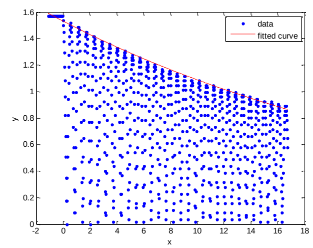
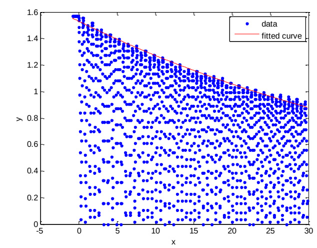
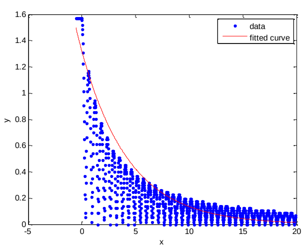
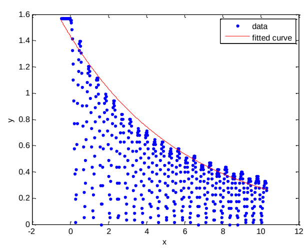
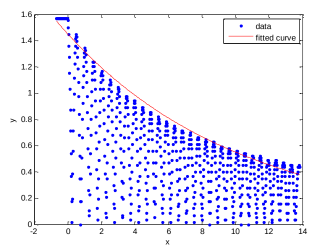
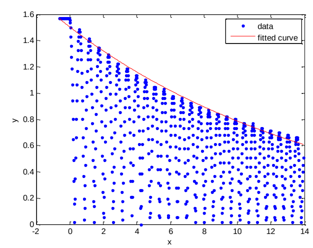
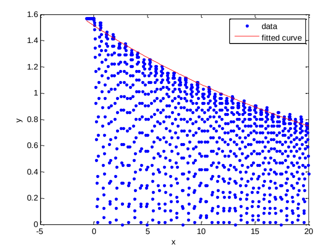
This is the full MATLAB script used to analyze the data and extract the α values from the collected data. It was run on the 2012b version of MATLAB and is saved under the name “PendulumDataAnalysis.m.”

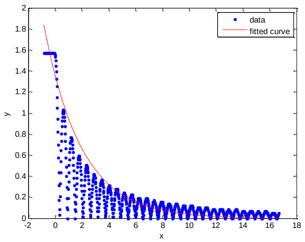
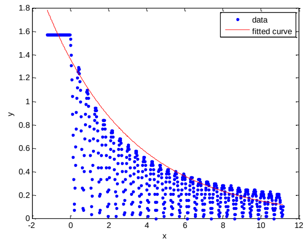
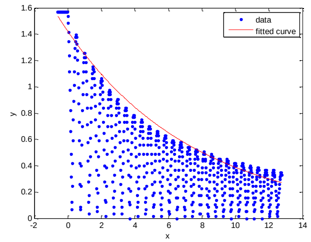
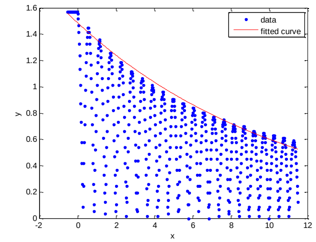
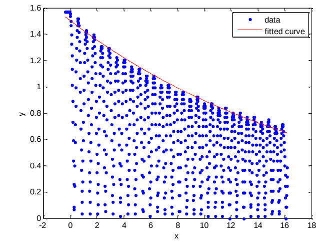
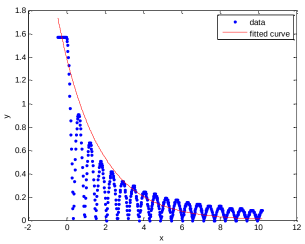
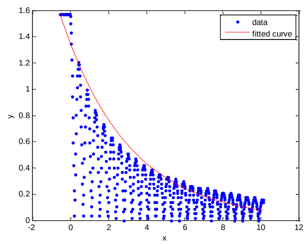
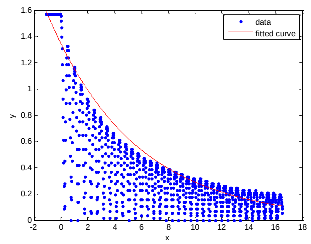
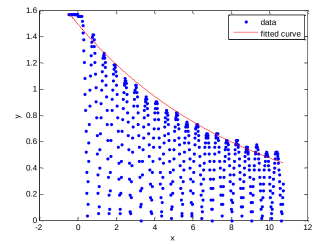
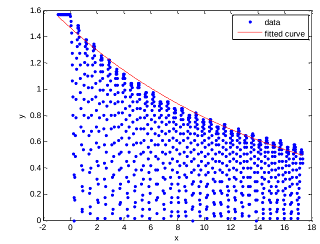
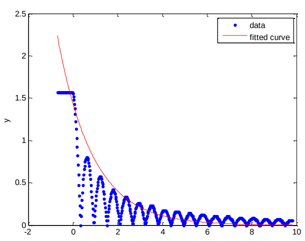
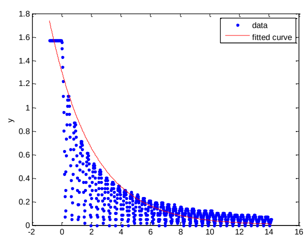
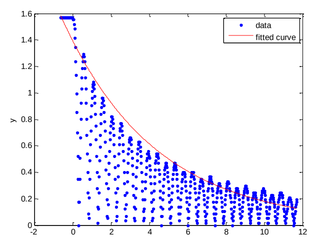
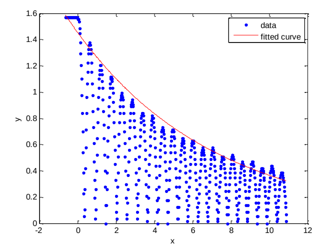
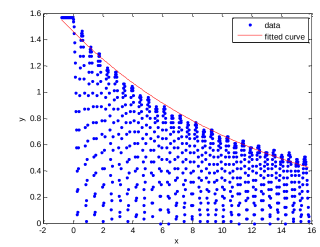
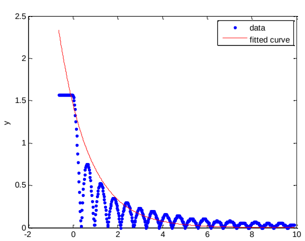
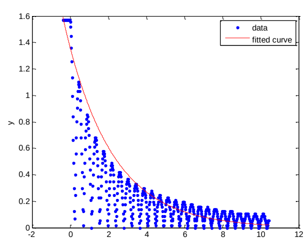
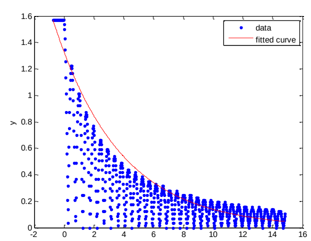
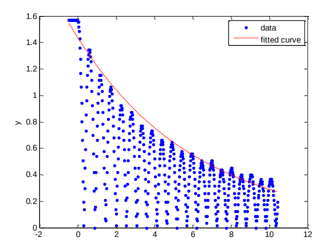
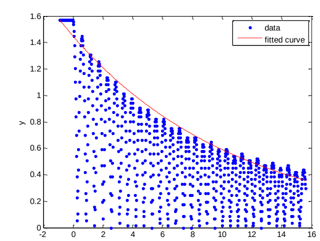
```
%Scott A Bell - Revision D 3/22/2016
%Experimental   Methods: MCET - 400
%Experiment 3 - Pendulum Damping
clc
clear

excelSheet = 'LowMOI.xlsx';
             'LowMidMOI.xlsx';
             'MidMOI.xlsx';
             'MidHighMOI.xlsx';
             'HighMOI.xlsx';

[status,sheets] = xlsfinfo(excelSheet);
sheetQuantity = numel(sheets);
samples_persecond = 50;
t_start = [];

 for i = 1:sheetQuantity;
   RawData = xlsread(excelSheet,i);
   x_prep = RawData(:,1);
   y_prep = -1*RawData(:,2);

   for j = 1:length(RawData(:,2))
       if y_prep(j) < 0
          t_start(i) = j-1;
          break
       end
   end

   t_delta = x_prep(j) - x_prep(1);

   x = (1/samples_persecond)*(x_prep - t_start(i));
   y = abs((y_prep + 90)*(pi/180));

   [b,a] = findpeaks(y);

   [f,g] = fit((a-t_delta)*(1/samples_persecond),b,'exp1');
   p = coeffvalues(f);
   alpha(i,1) = p(2);
   beta(i,1) = p(1);
   gamma(i,1) = g;
   plot(f,x,y)

 end
 coeff = [];
 coeff = [alpha,beta]
```

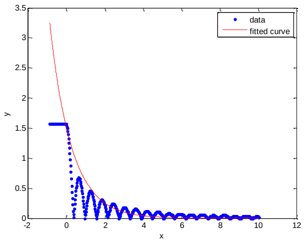
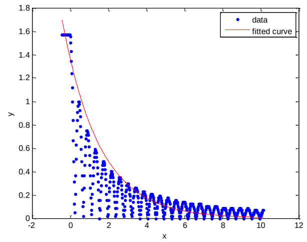
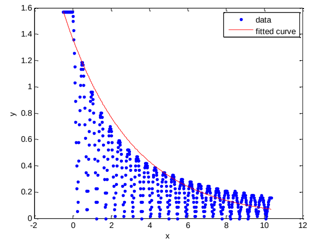
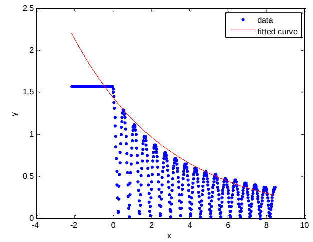
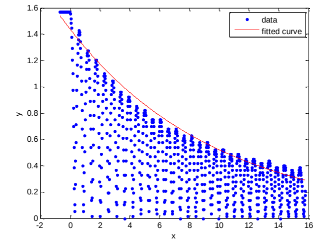
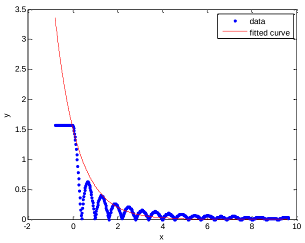
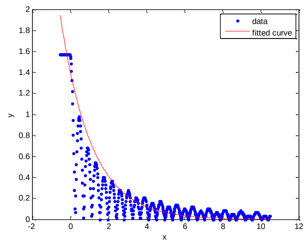
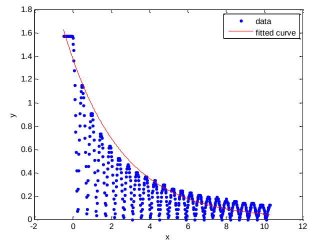
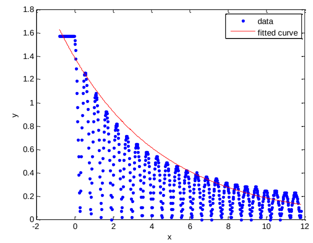
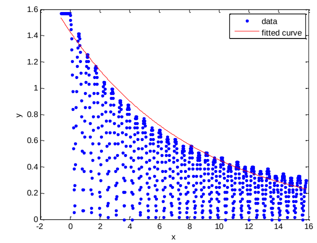
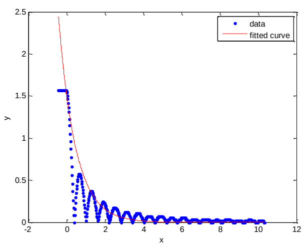
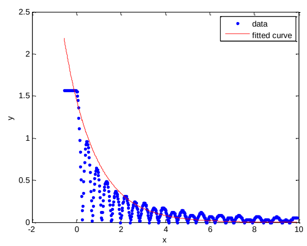
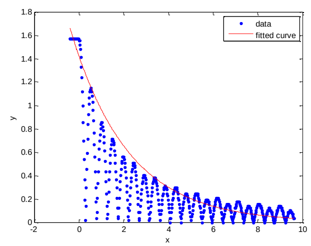
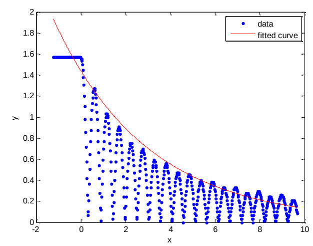
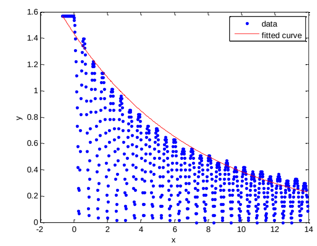
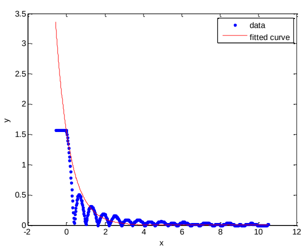
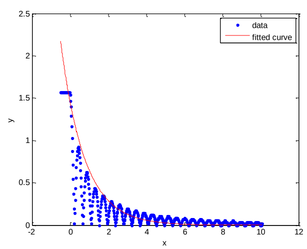
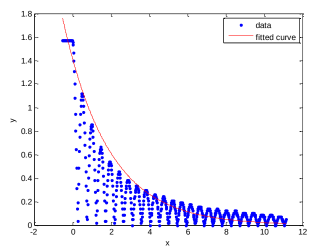
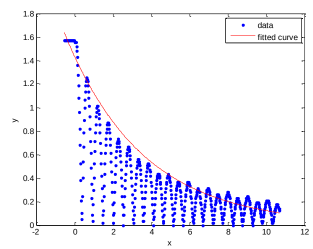
Additional information on the functionality and process of this script can be obtained from GD & T-REX.

APPENDIX C

This appendix contains plots of the "θ vs time" for all 55 test case scenarios. These plots also include the line of best fit as discussed in Part 2 of section 4.2.3. Again, the θ values are shown as blue dots. The maximums should be evident, and the red line that passes through these maximums is also included in the plots below. Also, the R^2 value as well as the extracted α value are included above every plot. Every row in the below array is the data collected from a specific damping card while every column is a specific moment of inertia. Raw Data (blue) is available upon request.

Card	MOI1: 0.002987 $kg*m^2$	MOI2: 0.008411 $kg*m^2$	MOI3: 0.015071 $kg*m^2$	MOI4: 0.024395 $kg*m^2$	MOI5: 0.044435 $kg*m^2$
None	R^2= .9939 α=.03309 	R^2= .9967 α=.01432 	R^2= .9960 α=.007091 	R^2=.9963 α=.004594 	R^2= .9624 α=.003189 
1	R^2= .9515 α=.1193 	R^2= .9892 α=.09219 	R^2= .9910 α=.05378 	R^2= .9949 α=.03414 	R^2= .9885 α=.01849 
2	R^2= .9397 α=.2271 	R^2= .9732 α=.1636 	R^2= .9802 α=.09680 	R^2= .9902 α=.06482 	R^2= .9875 α=.03511 

3	R^2= .9404 α=.3591 	R^2= .9573 α=.2207 	R^2= .9724 α=.1297 	R^2= .9847 α=.09032 	R^2= .9848 α=.05192 
4	R^2= .9405 α=.4762 	R^2= .9533 α=.2831 	R^2= .9588 α=.1532 	R^2= .9841 α=.1143 	R^2= .9814 α=.06263 
5	R^2= .9479 α=.6342 	R^2= .9394 α=.3449 	R^2= .9626 α=.2043 	R^2= .9774 α=.1376 	R^2= .9808 α=.07778 
6	R^2= .9511 α=.7384 	R^2= .9496 α=.4036 	R^2= .9502 α=.2257 	R^2= .9738 α=.1597 	R^2= .9778 α=.08963 

7	R^2= .9636 α=.8845 	R^2= .9468 α=.5120 	R^2= .9531 α=.2872 	R^2= .9746 α=.1971 	R^2= .9776 α=.1013 
8	R^2= .9645 α=1.003 	R^2= .9499 α=.6114 	R^2= .9565 α=.3405 	R^2= .9644 α=.2018 	R^2= .9755 α=.1148 
9	R^2= .9719 α=1.134 	R^2= .9647 α=.7210 	R^2= .9624 α=.3885 	R^2= .9732 α=.2381 	R^2= .9766 α=.1312 
10	R^2= .9749 α=1.349 	R^2= .9617 α=.8030 	R^2= .9626 α=.4181 	R^2= .9703 α=.2450 	R^2= .9784 α=.1413